# MATHEMATICAL THEORY OF EVIDENCE IN MARITIME TRAFFIC ENGINEERING

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#### Abstract

Nowadays operator at maritime traffic monitoring station is assumed to have access to a great amount of data. Information come from different sources and the data are generated by multiple of sensors. Multiple sources of data create challenge regarding data association. The challenge is met by data fusion. By means of fusion, different sources of information are combined to arrive at proper final decision. Ship's presence within a confined area defines a non-empty bounded closed interval. It can be denoted by the earliest and latest bounds of the closed time interval at a given possibility level. To assess situation within any confined region one should take into account total of safety factors of all ships present within forecast imprecise slots of time. Safety factors enable vessels' classification regarding potential consequences of an accident. In general approach environmentally dangerous freight and huge tonnage increase the factor. Safety factors are treated as fuzzy, imprecise values. Small ranges of values are assigned to small craft without dangerous cargo. The largest intervals are reserved for huge crude carriers. Associated data enable the VTS operator to approximate congestion for each restricted and considered as important areas. To forecast and assess situation within such areas all ships are to be identified and classified. The process usually involves uncertainty, ambiguity and partiality in available evidence. The new AIS technology itself causes ambiguity with respect to identification of crafts. Published statistics point at incorrect data transferred in the system. Therefore partial evidence is to be taken into account while identifying objects. Dempster-Shafer reasoning is helpful when combining evidence in order to refine objects. Situation in which one spotted new target and tries to find out what ship this could be is considered. Fuzzy evidence embraced within frame of discernment and related to this identification case will be assumed.

Keywords: sea transport, fuzziness, evidential theory, Dempster-Shafer fuzzy reasoning

### 1. Introduction

The operational areas of sea going vessels can be divided into three major parts: port, restricted area and open sea. Published statistics show that restricted areas create highest risk of collision and stranding. Within restricted areas there are zones of routes intersections, where potential collision manoeuvres are hampered. Such zones are of particular concern for those who are engaged in practical as well as theoretic aspects of risk reduction. There were many risk model developed all of them focused on probability of collision estimation. Most of them assume that the probability of collision depends on the crossing area topology as well as on an encounter rate [1]. An encounter is a situation involving penetration of the domain area of a ship by another vessel. Thus any method of distributing the traffic that results in the avoidance of a local accumulation of ships should be considered vital in restricted areas since it would lead to a reduction in the number of encounters. This paper deals with congestion avoidance problem by aiming at quantifying navigational situation within confined crossing routes areas. It is supposed to help in granting uneventful passages through a restricted area. Alternatively, based on obtained evaluation, traffic within an area can be allocated over the whole region. Proposed evaluation of the navigational situation deals with uncertainty, ambiguity and incomplete evidence.

### 2. Uncertainty and imprecision in traffic engineering

Uncertainty and ambiguity is related to human reasoning and judgment. Stochastic and epistemic or subjective uncertainties are selected and discussed in many papers. Stochastic also called aleatory uncertainty reflects unknown, usually unpredictable behavior of a system. The system behaves in stochastic way when its future states can be foreseen based on probability theory. In maritime traffic engineering there are acceptable alternatives routes exist quite often. Attempt to point at the route taken by particular vessel is related the aleatory uncertainty. Traditional statistical approaches are helpful in this respect. Data gathered in stored records are to be analyzed in order to draw final conclusion.

Shortage of knowledge or lack of evidence creates another kind of uncertainty. Epistemic or subjective uncertainty results from insufficient or vague evidence. Question of identity of a spotted object refers to this sort of uncertainty. It is quite often when observer at monitoring station spots new radar echo and tries to find out what kind of vessel this could be. Usually there is some evidence available, for instance radar echo signature and speed estimation could be helpful. Modern AIS technology transfers data useful in identification process but published statistics indicate errors in its functionality [7]. Yet another sources point at wide misuse of the technology, many ships carry transceivers which are simply switched off.

Radar screen delivers plenty of data used for objects identification. These data are further used for navigational situation refinement. Quantifying navigational condition within confined crossing routes areas is crucial from overall safety standards. Potential congestion creates threats that can be foreseen and avoided.

Questions involving epistemic uncertainty that refer to an identity of a vessel could be:

- What type of ship is associated with each of the echoes seen on the radar screen?
- What is a tonnage (expressed in linguistic terms) of each vessel?
- What hazardous cargo (if any) does each of the vessels carry?
- How much of dangerous cargo does each of the vessels carry? Aleatory uncertainty is imbedded in other issues, for example:
- What are time frames of passage through the intersection routes zone?
- Does the intended itinerary pass through close to the middle part of the crossing zone?

### 3. Probability theory and Mathematical Theory of Evidence

In classical probability theory the knowledge of the probability of an event is of primary importance. It can also be used to calculate likelihood of the contrary statement. In this approach if radar observer classifies a new spotted object as a big container carrier with a likelihood of 0.3, that mean that the expert believes that it is not this particular ship with probability of 0.7? There is no space for other events within the remaining value. This fact articulates the main drawback of the classical probability theory. It disables modeling uncertainty and partial evidence that appears in epistemic uncertainty. Probability theory is limited in its ability when dealing with sort of uncertainty.

It is limited in its universality due to requirement of complete data regarding probability of all considered events. Mathematical Theory of Evidence is more flexible in this aspect. Contrary to probability theory it enables modeling knowledge and ignorance. Evidence can be combined even partial knowledge associated with less meaningful facts may end up in valuable conclusions. In combining evidence credibility judgments are to be obtained for each problem domain hypothesis. Hypothesis refers to atomic and/or molecular events. Sometime atomic cases are beyond available scope of knowledge. At the same time reasoning can be made with respect to a structured or molecular event. New extensions to cope with imprecision are also available since it is often that to obtain precise figures is infeasible.

#### 4. Dempster-Shafer reasoning

There are three fundamental terms in Dempster-Shafer scheme of combination: the basic probability assignment (bpa), the belief function (bel), and the plausibility function (pla) [6]. The term "basic probability assignment" defines a mapping of events or hypotheses to a value within range from 0 to 1. The bpa for a given hypotheses A, expresses the relative strength of this hypothesis. The strength is usually abbreviated as m(A) and is called as mass of evidence

attributed to the event A. Each of hypotheses (atomic or molecular) is assumed to have assigned a piece of evidence by information provider. The provider is human or computer expert, for reasoning there are various data sources exploited.

In the approach a set  $\Omega$  consisted of several hypotheses creates a frame of discernment. The hypotheses are elements of this set. For problem of identifying unknown target spotted on radar display the frame might contains:  $\Omega = (,, \text{container carrier''}, ,, \text{bulk carrier''}, ,, \text{crude oil carrier''}, ,, LPG carrier''). Based on available evidence the spotted target is supposed to be one the mentioned ships. The example set consisted of four elements (objects) is usually written in abbreviated form as <math>\Omega = (s_1, s_2, s_3, s_4)$ . Power set  $2^{\Omega}$  is a set of all subsets of  $\Omega$  in form of single and conjunctions of hypotheses also called atomic or molecular events. For the example set  $\Omega$  one has  $2^{\Omega} = (\emptyset, (s_1), (s_2), (s_3), (s_4), (s_1, s_2), (s_1, s_3), (s_1, s_4), (s_2, s_3), (s_2, s_4), (s_3, s_4), (s_1, s_2, s_3, s_4))$ . Each of the hypotheses (atomic or molecular) can be assigned a piece of evidence m(A) with  $A \in 2^{\Omega}$  by information provider. Sometime available data are crisp figures, for example the vessel is 50 000 dwt bulk carrier. Quite often generality of reasoning requires subjective assessment like ,,small'', ,,large'' etc. Typical for human reasoning is rather ,,large bulk carrier'' instead of ,,50000 dwt bulk carrier''. Question whether 50000dwt means ,,large'' is to be subjectively answered.

Dempster-Shafer theory proved to be the most popular method, in evidential reasoning [6]. However, it is very often that we need to face the problem of making decisions in situation when available information is not only uncertain but also imprecise and vague, such as "the vessel is large" and "the cargo is mildly hazardous". Such kind of information is difficult to be processed by basic DS theory, but is the primary concern of fuzzy logic. Therefore, the combination of DS theory and fuzzy set theory is a good way to solve complex problems that include fuzzy information. Extended DS theory is capable to process both crisp and fuzzy data.

In many practical frameworks "imprecise masses of evidence are assigned to fuzzy propositions". In combining fuzzy evidence there are two kinds of information involved:

- possibility of occurrence of the event ( $F_i$ ) given as membership functions of the linguistic terms used for qualification of the event, the function is abbreviated as  $\mu_{Fi}(x)$ ,
- membership functions of the linguistic terms used in event assessment, for event (*A*) we have  $\mu_A(x)$ . Simplified interval-based data is used in problems involving ambiguity. More general fuzzy approach exploits intervals at selected possibility levels also called as  $\alpha$ -cuts. Suppose there is an unidentified object that should be classified as: very large, large, medium, small or very small. There are some sources of data to extract the judgment. Usually it is an expert role to deliver their opinion based on available evidence. Expert expresses his credibility regarding given proposition along with his doubtfulness in this respect. Uncertainty is assigned to  $\Omega$  set, which in crisp case is called frame of discernment. In case of fuzzy model it is rather called possibility space. Nevertheless meaning of it expresses statement "everything is possible".

Let us consider example in which first expert states that the object is large vessel (L). He attributes interval mass of evidence  $m_1(L) = [0.4, 0.8]$  to his statement. He also is not very much doubtful on his opinion so he assumes  $m_1(\Omega) = [0.2, 0.4]$  that this could be yet another ship. Second expert's opinion is a bit different. He is rather convinced the object is a medium vessel (M) with mass of evidence  $m_2(M) = [0.6, 0.9]$ . His uncertainty specifies assignment to  $\Omega$  set;  $m_2(\Omega) = [0.2, 0.4]$ .

### 5. Safety Factors

Traffic is classified taking into account gross tonnage of a vessel and a kind of cargo she has on board. Safety factors have been introduced to enable classification. In general approach environmentally dangerous freight and huge tonnage increase the factor. As it was proposed the factor vary on an integer scale such that the higher the number the more serious the consequences of an accident. There was range from 1 to 10 suggested by the author [3]. Small value was assigned to small craft without dangerous cargo. The largest value was reserved for huge crude carriers. It was assumed that safety factors imprecise, fuzzy values. Suggested assignment of imprecise safety factors to selected classes of crafts is presented in Tab. 1.

General scheme of assignment is based on five classes of ship's tonnage: very small, small, medium, large and very large. There were three categories of cargo: mildly dangerous (MD), dangerous (D) and very hazardous (VD). Quantity of cargo was classified using the same terms as for ship's tonnage. Tab. 1 contains k value to be used with formula 1 in order to calculate a fuzzy factor.

		Quality of cargo										
	Ν	/ildly d	angeroi	15		Dangerous			Very Dangerous			
		Cargo d	quantity	r		Cargo quantity			Cargo quantity			
Tonnage	small	medium	large	very large	small	medium	large	very large	small	medium	large	very large
very small	1	2	3	4	5	6	7	8	9	10	11	12
small	13	14	15	16	17	18	19	20	21	22	23	24
medium	25	26	27	28	29	30	31	32	33	34	35	36
large	37	38	39	40	41	42	43	44	45	46	47	48
very large	49	50	51	52	53	54	55	56	57	58	59	60

Tab. 1. Values of k for extended fuzzy safety factors assignment

Fuzzy Safety Factor evaluation involves subjective evaluation of ships tonnage and amount and quality of their hazardous cargo see Fig. 1. At first vessel's tonnage is to be subjectively classified then the same must be done regarding her freight.

$$SF_{k} = \begin{cases} [0, 0, w_{T} / 2^{*} w, 0.5^{*} w] & \text{if } k = 1 \\ [(k-1)^{*} w, k^{*} w - w_{T}^{*} w, k^{*} w + w_{T}^{*} w, (k+1)^{*} w] & \text{if } 1 < k < n_{c} \\ [1-0.5^{*} w, 1-w_{T} / 2^{*} w, 1, 1] & \text{if } k = n_{c} \end{cases}$$
(1)

where:

 $w = 1/(n_c-1)$ ,  $n_c = n_T * n_H * n_{QH}$  - product of tonnage terms and hazardous cargo quantity and quality classes,  $w_T \in [0, 1]$  - trapezoid factor. Note that  $w_T = 0$  means that SF<sub>k</sub> is a triangular fuzzy value and  $w_T = 1$  means rectangular one.

6. Membership functions

Membership functions in fuzzy events are considered subjectively. In many cases such functions are arbitrary selected regular ones, for example trapezoids for above mentioned Safety Factors. There are also membership functions created based on experts opinions. Basic for these functions are belonging frequency for unity interval  $x_i$ . Let us consider statistical experiment in which experts are asked what they think about 40000 dwt tonnage of a vessel in terms "very small", "small", "medium", "large", "very large". It is also assumed that experts are aware of local conditions. Ship of this tonnage is decisively medium or even small at open sea but may be perceived as large or very large within confined region. Experts opinions are gathered in Tab. 2, result membership function is in the last row of the table.

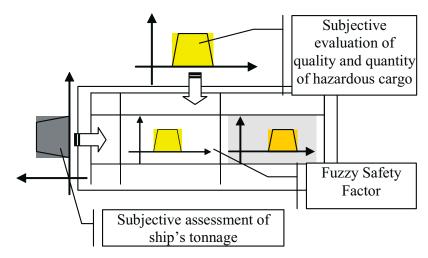


Fig. 1. Fuzzy Safety Factor assignment involves subjective evaluation of ships tonnage and hazardous cargo

		linguistic term														
	ve	ry sm	all		sm	all		med	lium		la	rge		ve	ry larg	<i>g</i> e
Expert	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1									х	х	х	х	х			
2							х	х	х	х	х					
3								х	Х	х	х	х				
4										х	х	х	х			
5											х	х				
μ	0	0	0	0	0	0	0.2	0.4	0.6	0.8	1	0.8	0.4	0	0	0

Tab. 2. Meaning of "40000 dwt ship" delivered by experts

Membership function of the sentence "how large is 40 000 dwt ship" for the above experiment is written as:  $\mu_{(40)}(x_i) = (0.2/7, 0.4/8, 0.6/9, 0.8/10, 1/11, 0.8/12, 0.4/13)$ 

Similar experiments for 60 000, 25 000 and 4 000 dwt yielded in:

- $\mu_{(60)}(x_i) = (0.2/8, 0.4/9, 0.8/10, 1/11, 1/12, 0.6/13, 0.4/14),$
- $\mu_{(25)}(x_i) = (0.2/3, 1/4, 1/5, 0.8/6, 0.2/7),$
- $\mu_{(4)}(\mathbf{x}_i) = (0.2/3, 1/4, 1/5, 0.8/6, 0.2/7).$

Figures, elements of the sets, are relative frequencies given for specified unit interval. Zero values are usually omitted. Presented functions will be further used in discussed example.

Let us carry out yet another statistical experiment in which experts are asked what they think about 190 m length of a vessel in above specified terms. As before local conditions should be taken into account by experts. Membership function of the sentence "how large is 190 m ship" as a result of the experiment can be written as:

-  $\mu_{(190m)}(x_i) = (0.2/8, 0.6/9, 0.8/10, 1/11, 1/12, 0.4/13, 0.2/14)$ 

## 7. Fuzzy data combination - practical case

Let us consider case in which VTS radar operator tries to classify new spotted object. Using his radar he estimated objects speed as about 18 knots, at the same time he is not sure about updated tidal streams and current in the region. His AIS receiver reads that the new object length is 190 m. At the other side he has a list of expected traffic with data collected in Tab. 3. Having all the data he is supposed to issue his opinion regarding tonnage of the spotted vessel. Refined data and

observer subjective judgments are collected in table 4. Final data used in combination presents Tab. 5.

Ship	Ship Tonnage dwt/length		Quantity of hazardous cargo	Maximum speed
s <sub>1</sub> - container carrier	$s_1$ - container carrier 40 000/185		10 containers stowed on deck	20 kt
s <sub>2</sub> - crude oil tanker	60 000/190	Crude oil	15 000 tons	14 kt
s <sub>3</sub> - bulk carrier	25 000/150	Grain - non dangerous	20 000 tons of grain	13.5 kt
s <sub>4</sub> - LPG carrier	4 000/100	Liquefied petroleum gas	3 000 tons	18 kt

Tab. 3. Data related to expected vessels

Tab. 4. Available evidence and possible subjective judgments

Report	Subjective assessment referring to particular ship level of uncertainty	Subjective assessment of class of cargo	Subjective assessment of quantity of hazardous cargo
Preliminary estimation of the object's speed is about 18 kt. It means that one should consider whole set of: (s <sub>1</sub> , s <sub>2</sub> , s <sub>3</sub> , s <sub>4</sub> ) with high confidence	container carrier - very good bulk carrier - poor tanker - poor LPG tanker - good uncertainty - low	dangerous uncertainty - low	small or medium uncertainty - low
Detected object is at range of about 20 Nm, radar echo signature is strong and clear. It means that most likely large vessel has been spotted	container or bulk carrier or tanker - fair uncertainty - low	dangerous or normal uncertainty - very low	very small, small or medium uncertainty - low
AIS reads 190 m length. Due to known statistics reliability of the source is fair	tanker - fair uncertainty - fairly convinced	data not available	data not available

Scale of ship qualification includes following set of terms: "very poor" (0, 0, 0.2), "poor" (0, 0.2, 0.4), "fair" (0.2, 0.4, 0.6), "good" (0.4, 0.6, 0.8), "very good" (0.6, 0.8, 1), "excellent" (0.8, 1, 1). Scale of uncertainty qualification embraces: "very low" (0, 0, 0.2), "low" (0, 0.2, 0.4), "fairly convinced" (0.2, 0.4, 0.6), "convinced" (0.4, 0.6, 0.8), "very convinced" (0.6, 0.8, 1), "sure" (0.8, 1, 1).

Triples embraced in parenthesis are regular, triangular fuzzy numbers. Middle figure is a core of the number, the first and last create so called support. For (0.2, 0.4, 0.6) core is equal 0.4 and support is a range [0.2, 0.6]. According to the fuzzy sets theory support is  $\alpha$ -cut for  $\alpha$  possibility level equal to 0. Results of fuzzy combination presented in tables were obtained for supports ( $\alpha = 0$ ). Details regarding method of combination can be found in [2]. Calculations were carried out using software downloaded from: http://www.hds.utc.fr/~tdenoeux/perso/doku.php?

Results of fuzzy evidence combination are gathered in Tab. 6. The highest credibility is associated with membership functions similar to those related to 40 000 or 60 000 dwt ships (see Tab. 2). Combined evidence is fundamental for answering different queries regarding spotted object refinement. One likes to find out existing support for object's features. Namely it is interesting how much support exist to classify an object as medium vessel or a large one.

Having given pattern one asks for credibility of another proposition. In this example it is crucial to calculate for given resulting sets of combination, as presented in Tab. 6, a belief and possibly plausibility that the spotted object is a large or a medium vessel. Results for the queries regarding terms large and medium in context of data embraced in Tab. 6 are presented in Tab.  $7^1$ .

Report	Membership functions for subjective assessment of tonnage	Subjective quality of case selection and its fuzzy number
	$\mu_{(40)} = (0.2/7, 0.4/8, 0.6/9, 0.8/10, 1/11, 0.8/12, 0.4/13)$	very good (0.6, 0.8, 1)
	$\mu_{(60)} = (0.2/8, 0.4/9, 0.8/10, 1/11, 1/12, 0.6/13, 0.4/14)$	poor (0, 0.2, 0.4)
1	$\mu_{(25)} = (0.2/5, 0.6/6, 1/7, 1/8, 0.6/9, 0.4/10)$	poor (0, 0.2, 0.4)
_	$\mu_{(4)} = (0.2/3, 1/4, 1/5, 0.8/6, 0.2/7)$	good (0.4, 0.6, 0.8)
	$\mu_{(any)} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$	low (0, 0.2, 0.4)
2	$\begin{array}{c} \mu_{(40)} \lor \mu_{(60)} \lor \mu_{(25)} = \\ (0.2/3, 1/4, 1/5, 0.8/6, 0.2/7, 0.4/8, 0.6/9, 0.8/10, 1/11, 1/12, 0.6/13, \\ 0.4/14) \end{array}$	fair (0.2, 0.4, 0.6)
	$\mu_{(any)} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$	fairly convinced (0.2, 0.4, 0.6)
3	$\mu_{(190m)} =$ (0.2/8, 0.6/9, 0.8/10, 1/11, 1/12, 0.4/13, 0.2/14)	fair (0.2, 0.4, 0.6)
	$\mu_{(any)} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$	fairly convinced (0.2, 0.4, 0.6)

Tab. 5. Evidential data regarding forecast traffic and their assessment

### Tab. 6. Results of fuzzy evidence combination

Resulting membership function	Fuzzy belief
(0, 0, 0, 0, 0, 0, 0, 0.2, 0.6, 0.8, 1, 0.8, 0.4, 0, 0, 0)	[0.12, 0.36]
(0, 0, 0, 0, 0, 0, 0.2, 0.4, 0.6, 0.8, 1, 0.8, 0.4, 0, 0, 0)	[0.12, 0.36]
(0, 0, 0, 0, 0, 0, 0, 0, 0.2, 0.6, 0.4, 0, 0, 0, 0, 0, 0)	[0, 0]
(0, 0, 0, 0, 0.2, 0.6, 0.2, 0.4, 0.6, 0.4, 0, 0, 0, 0, 0, 0)	[0, 0.144]
(0, 0, 0, 0, 0.2, 0.6, 1, 1, 0.6, 0.4, 0, 0, 0, 0, 0, 0)	[0, 0.144]
(0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0, 0, 0, 0, 0, 0, 0, 0)	[0.08 0.24]
(0, 0, 0.2, 1, 1, 0.8, 0.2, 0.1, 0, 0, 0, 0, 0, 0, 0, 0)	[0.08 0.24]
(0, 0, 0, 0, 0, 0, 0, 0.2, 0.6, 0.8, 1, 1, 0.4, 0.2, 0, 0)	[0, 0]
(0, 0, 0.2, 1, 1, 0.8, 0.2, 0.4, 0.6, 0.8, 1, 1, 0.6, 0.4, 0, 0)	[0, 0.144]
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	[0, 0.144]

Tab. 7. Results of queries regarding magnitude of the spotted object

magnitude of a vessel	pattern membership function	fuzzy belief	fuzzy plausibility
large	(0.5/10, 1/11, 1/12, 0.5/13)	[0.17, 0.49]	[0.35, 0.84]
medium	(0.5/7, 1/8, 1/9, 0.5/10)	[0.08, 0.35]	[0.38, 0.64]

<sup>&</sup>lt;sup>1</sup> Results of fuzzy combination presented above were obtained for cores ( $\alpha$ =1) of the fuzzy values. Calculations were carried out using software downloaded from: http://www.hds.utc.fr/~tdenoeux/perso/doku.php?

### 8. Example of navigational situation refinement

To assess situation within confined areas approximations regarding all scheduled traffic are to be estimated. As an example let us consider situation presented in Fig. 2.

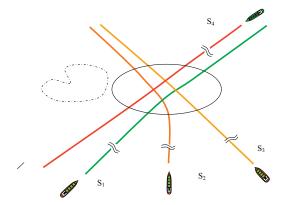


Fig. 2. Example of four vessels due to encounter within confined area

There are four vessels that are very likely to encounter within the crossing routes area. The vessels were identified as: small with small quantity of dangerous cargo S&S&D, medium with large quantity of dangerous cargo M&L&D and small with small amount of dangerous cargo S&S&MD, the last one is large container with large amount of dangerous cargo onboard L&L&D. Data regarding involved ships are gathered in table 8. Consecutive columns in the table contain:

- abbreviated ship characteristic,
- k factor extracted from table 1,
- Safety Factor calculated with formula 1 for given k,
- [Belief<sup>+</sup>, Plausibility<sup>+</sup>] maximal limits of credibility attributed to the identification process,
- $f_{si}(m)$  "staying within an area" membership function value.

Ship	k	Safety Factor	[Belief <sup>+</sup> , Plausibility <sup>+</sup> ]	f <sub>si</sub> (m)
S&S&D	17	(0.271, 0.281, 0.295, 0.305)	[0.40, 0.60]	1,0
M&L&D	31	(0.508, 0.519, 0.532, 0.542)	[0.55, 0.85]	1,0
S&S&D	13	(0.203, 0.214, 0.227, 0.237)	[0.40, 0.60]	0,8
L&L&D	43	(0.712, 0.722, 0.736, 0.746	[0.37, 1.00]	0,9

Tab. 8. Data regarding ships mentioned in the example

The normalized result figures for selected possibility levels are given in Tab. 9. The figures present fuzzy evaluation of navigational situation within crossing area within time frame in which all the considered vessels will somewhere in the vicinity. Given numbers are left  $(L^{\alpha})$  and right  $(R^{\alpha})$  boundary values for specified  $\alpha$  - cuts.

Tab. 9. Final normalized fuzzy evaluation of navigational situation within crossing area

α	Lα	R <sup>α</sup>
0,0	0.485	1.000
0,2	0.487	0.996
0,4	0.489	0.992
0,6	0.492	0.988
0,8	0.494	0.984
1,0	0.496	0.979

## 9. Conclusions

Problem of evaluation of the navigational situation within crossing routes area was presented. Given an evaluation for each node of a route one is able to recommend the best passage option. Option means intended itinerary (if alternatives exist) or passage time frame. The goal aims at minimizing congestions, subsequently at reducing risk of marine accident, within naturally confined areas. Navigational situation within restricted regions is characterized using fuzziness. Ships fuzzy safety factors related to gross tonnage as well as amount and sort of carried cargo were used for evaluation. Presence within confined area was also considered as fuzzy set. Membership function learning method was discussed by the author in his previous paper [4]. Arrival and departure from selected routes crossing areas are trapezoidal imprecise values. Membership functions are to be learned for particular region, weather condition and each class of vessels [5].

Having at his disposal reliable fused large quantity of data and appropriate software tools VTS operator seems to be able to solve rather complicated problem of issuing advices on best passage for selected classes of ships particularly those constrained by draught with hazardous cargo onboard. The decision making problem was considered by the author in his previous paper [3].

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